UNIVERSITY OF DELHI

MASTER OF SCIENCE in MATHEMATICS

(MMATH)

(Effective from Academic Year 20XX -XX)

PROPOSED SYLLABUS

M.Sc. Mathematics Revised Syllabus as approved by Academic Council on XXXX, 2018 and Executive Council on YYYY, 2018

CONTENTS

I. About the Department

The Department of Mathematics, University of Delhi took birth in 1947, with Prof. Ram Behari, an eminent Differential Geometer, as Head of the Department. Several distinguished mathematicians have been part of the Department's long and successful journey so far, Prof. R. S. Varma and Prof. U.N. Singh among them. While research activities in Operations Research, Information Theory, Coding Theory, Space Dynamics and Complex Analysis blossomed under the former's leadership, the latter made the Department a very strong, global research hub for Functional Analysis, Operator Theory and Harmonic Analysis. As the activities of the Department grew exponentially, in 1973, the single department metamorphosed into the Faculty of Mathematical Sciences with four constituent departments: Mathematics, Operations Research, Statistics and Computer Science. The South Campus unit of the Department of Mathematics also came into being in 1973. The Department of Mathematics continues to uphold the high traditions of teaching and research which have shaped it from the very beginning.

The Mathematics Department offers a masters programme in Mathematics besides two research programmes, the M.Phil. and Ph.D. in Mathematics. Nearly hundred students have been awarded Ph.D. degrees in the last five years. The department is also actively involved in administering the University's undergraduate programmes in Mathematics. We are supported by grants from DST(FIST), DST(PURSE) and UGC-DRS(SAP). Our excellent and highly experienced faculty, with qualifications from premier institutions and expertise in diverse fields including Algebraic Geometry, Coding Theory, Complex Analysis, Commutative Algebra, Combinatorics, Control Theory, Differential Equations, Dynamical Systems, Ergodic Theory, Field Theory, Fluid Dynamics, Functional Analysis, Harmonic Analysis, Operator Theory, Operator Algebras, Optimization and Topology, has helped us in securing high positions in various national and international rankings. For example, we are ranked 10 in Times Higher Education Ranking in India and 144 in Asia in 2018. Several faculty members are or have been fellows of prestigious national science academies and recipients of awards of excellence from such academies. Many members are also actively involved as consultant/advisor for UGC, DST, CSIR, UPSC, Lok Sabha and are on advisory committees of several universities in the country.

The M.Sc. Mathematics programme, aims to build strong foundations in core areas of higher mathematics in both the pure and applied areas. It is meant for students who would typically take up careers involving mathematical research or mathematical skills – in academia or in industry. The training imparted to the students helps them master the art of problem solving, developing logical reasoning and computational capabilities which are essential traits in all walks of life. Additionally, the knowledge of mathematical modeling and computational training which the students acquire during the programme makes them highly sought after. In keeping with the demands of industry and academia, the syllabus is updated regularly, with inputs taken from various stakeholders including students, alumni and parents at different stages of the preparation of the syllabus.

II. Introduction to CBCS (Choice Based Credit System)

Scope:

The CBCS provides an opportunity for the students to choose courses from the prescribed courses comprising core, elective/minor or skill-based courses. The courses can be evaluated following the grading system, which is considered to be better than the conventional marks system. Grading system provides uniformity in the evaluation and computation of the Cumulative Grade Point Average (CGPA) based on student's performance in examinations which enables the student to move across institutions of higher learning. The uniformity in evaluation system also enable the potential employers in assessing the performance of the candidates.

Definitions:

(i) 'Academic Programme' means an entire course of study comprising its programme structure, course details, evaluation schemes etc. designed to be taught and evaluated in a teaching Department/Centre or jointly under more than one such Department/ Centre.

(ii) 'Course' means a segment of a subject that is part of an Academic Programme

(iii) 'Programme Structure' means a list of courses (Core, Elective, Open Elective) that makes up an Academic Programme, specifying the syllabus, Credits, hours of teaching, evaluation and examination schemes, minimum number of credits required for successful completion of the programme etc. prepared in conformity to University Rules, eligibility criteria for admission.

(iv) 'Core Course' means a course that a student admitted to a particular programme must successfully complete to receive the degree and which cannot be substituted by any other course.

(v) 'Elective Course' means an optional course to be selected by a student out of such courses offered in the same or any other Department/Centre

(vi) 'Open Elective' means an elective course which is available for students of all programmes, including students of same department. Students of other Department will opt these courses subject to fulfilling of eligibility of criteria as laid down by the Department offering the course.

(vii) 'Credit' means the value assigned to a course which indicates the level of instruction; One-hour lecture per week equals 1 Credit, 2 hours practical class per week equals 1 credit. Credit for a practical could be proposed as part of a course or as a separate practical course

(viii) 'SGPA' means Semester Grade Point Average calculated for individual semester.

(ix) 'CGPA' is Cumulative Grade Points Average calculated for all courses completed by the students at any point of time. CGPA is calculated each year for both the semesters clubbed together.

(x) 'Grand CGPA' is calculated in the last year of the course by clubbing together of CGPA of two years, that is, four semesters. Grand CGPA is being given in Transcript form. To benefit the student a formula for conversation of Grand CGPA into %age marks is given in the Transcript.

III. M.Sc. Mathematics Programme Details:

Programme Objectives (POs):

The M.Sc. Mathematics programme's main objectives are

- To inculcate and develop mathematical aptitude and the ability to think abstractly in the student.
- To develop computational abilities and programming skills.
- To develop in the student the ability to read, follow and appreciate mathematical text.
- Train students to communicate mathematical ideas in a lucid and effective manner.
- To train students to apply their theoretical knowledge to solve problems.
- To encourage the use of relevant software such as MATLAB and MATHEMATICA.

Programme Specific Outcomes (PSOs):

On successful completion of the M.Sc. Mathematics programme a student will

- Have a strong foundation in core areas of Mathematics, both pure and applied.
- Be able to apply mathematical skills and logical reasoning for problem solving.
- Communicate mathematical ideas effectively, in writing as well as orally.
- Have sound knowledge of mathematical modeling, programming and computational techniques as required for employment in industry.

Programme Structure:

The M.Sc. Mathematics programme is a two-year course divided into four semesters. A student is required to complete at least 80 credits for the completion of the course and the award of degree. Of these, 40 credits have to be earned from Core Courses and 40 from Electives (not including open electives).

	SEMESTER	SEMESTER
PART-I (FIRST YEAR)	Semester I	Semester II
PART-II (SECOND YEAR)	Semester III	Semester IV

Course Credit Scheme:

Semester	Core Courses			Elective Courses			Open Elective Courses			Total Credits
	No. of papers	Credits (L+T/P)	**Total Credits**	No. of papers	Credits (L+T/P)	**Total Credits**	No. of papers	Credits (L+T/P)	**Total Credits**	
I	04	05	**20**	Nil	Nil	**Nil**	Nil		**Nil**	**20**
II	04	05	**20**	Nil	Nil	**Nil**	Nil		**Nil**	**20**
III	Nil			04	05	**20**	01	02	**02**	**22**
IV	Nil			04	05	**20**	01	02	**02**	**22**
Maximum Credits including Open Elective offered by the other departments			**40**			**40**			**04**	**88** **(4 Credits from open electives of other departments)**
Minimum Credits Required			**40**			**40**				**80 + 04** **(4 Credits from open electives)**

* For each Core and Elective Course there will be 4 lecture hours and 1 tutorial hour per week.
* Students who have done Bachelor degree in any subject and have studied at least one course of one year duration or two courses of one semester duration in Mathematics are eligible to choose Open Elective Courses offered by the Department of Mathematics.
* Open Elective Courses can be chosen leading to the maximum total of 8 credits.

Selection of Elective Courses:

Under each Elective course a student may choose one course from a basket of three or four options being offered by the Department. In case a particular course is over-subscribed, merit in the previous semester(s) examination(s) will be used to determine course allocations.

Teaching:

The Department of Mathematics is primarily responsible for organizing lecture work for M.Sc. Mathematics programme. Faculty from some other departments and constituent colleges may also be associated with lecture and tutorial work in the department.

Eligibility for Admissions:

Admission is done via two modes – entrance and merit with equal numbers of seats allotted to both modes. The details are as follows:

Mode	Category	Eligibility: Course requirement	Eligibility: Marks requirement
Entrance	1	Bachelor degree in any subject and has studied at least 3 courses each of one year duration or 6 courses each of one semester duration in Mathematics	50% marks in aggregate
Merit	2	B.A./B.Sc. (Hons.) Mathematics degree of University of Delhi	60% marks in aggregate

Assessment of Students' Performance and Scheme of Examinations:

1. English shall be the medium of instruction and examination.
2. All assessment will be based on Learning Outcomes for the course.
3. Duration of examination of each paper of 5 credits shall be 3 hours while that of 2 credits will be 2 hours.
4. Each Core and Elective (Open Elective offered by Department of Mathematics) paper will be of 100 (50) marks with two components namely internal assessment for 30 (15) marks and end semester examination for 70 (35) marks.

Pass Percentage & Promotion Criteria:

Pass Percentage: 40% or equivalent grade (as per University rules) in each course. A student must score the minimum pass marks in **each** of the Core and Elective courses to be awarded the degree.

Part I to Part II Progression:

For promotion to Part II, a student must have passed in at least four of the core courses of Part I.

Policy for Reappearance:

A student who has to reappear in a paper prescribed for Semester I/III may do so only in the odd Semester examinations to be held in November/December. A student who has to reappear in a paper prescribed for Semester II/IV may do so only in the even Semester examinations to be held in April/May.

Conversion of Marks into Grades: As per University Rules

Grade Points: Grade point table as per University Examination rule

CGPA Calculation: As per University Examination rule.

SGPA Calculation: As per University Examination rule.

Grand SGPA Calculation: As per University Examination rule.

Conversion of Grand CGPA into Marks:

As notified by competent authority the formula for conversion of Grand CGPA into arks is: Final percentage of marks = CGPA based on all four semesters × 9.5.

Division of Degree into Classes: As per University rules.

Attendance Requirement: 66% attendance in lectures required to appear for the end semester examination while 50% attendance for the in-house examination.

Span Period:

No student shall be admitted as a candidate for the examination for any of the Parts/Semesters after the lapse of **four** years from the date of admission to the Part-I/Semester-I of the M.Sc. Mathematics Programme.

Guidelines for the Award of Internal Assessment Marks:

30 (15) marks are allocated for Internal Assessment in each Core and Elective (Open Elective offered by Department of Mathematics) paper. Of this, 20 (10) marks will be based on in-house examinations, with a common question paper across sections, while 10 (5) marks are for evaluation by individual teachers in each Core and Elective (Open Elective offered by Department of Mathematics) paper. This evaluation may be done by the teacher via class tests, assignment, presentations, viva-voce etc.

Semester Wise Details:

SEMESTER I				
Number of Core Courses	Four			
	Credits in each Core Course			
Courses	Theory	Practical	Tutorial	Credits
Core Course 1	04	-	01	05
Core Course 2	04	-	01	05
Core Course 3	04	-	01	05
Core Course 4	04	-	01	05
Total credits in Core Courses	20			
Number of Elective Courses	Nil			
Total credits in Elective Courses	Nil			
Number of Open Elective Courses	Nil			
Total credits in Open Elective Courses	Nil			
Total credits in Semester I	**20**			

SEMESTER II				
Number of Core Courses	Four			
	Credits in each Core Course			
Courses	Theory	Practical	Tutorial	Credits
Core Course 5	04	-	01	05
Core Course 6	04	-	01	05
Core Course 7	04	-	01	05
Core Course 8	04	-	01	05
Total credits in Core Courses	20			
Number of Elective Courses	Nil			
Total credits in Elective Courses	Nil			
Number of Open Elective Courses	Nil			
Total credits in Open Elective Courses	Nil			
Total credits in Semester II	**20**			

SEMESTER III				
Number of Core Courses	Nil			
Total credits in Core Courses	Nil			
Number of Elective Courses	Four			
	Credits in each Elective Course			
Courses	Theory	Practical	Tutorial	Credits
Elective Course 1	04	-	01	05
Elective Course 2	04	-	01	05
Elective Course 3	04	-	01	05
Elective Course 4	04	-	01	05
Total credits in Elective Courses	20			
Number of Open Elective Courses	One			
	Credits in each Open Elective Course			
Course	Theory	Practical	Tutorial	Credits
Open Elective Course 1	02	-	-	02
Total credits in Open Elective Course	02			
Total credits in Semester III	**20 + 2 = 22***			

SEMESTER IV				
Number of Core Courses	Nil			
Total credits in Core Courses	Nil			
Number of Elective Courses	Four			
	Credits in each Elective Course			
Courses	Theory	Practical	Tutorial	Credits
Elective Course 5	04	-	01	05
Elective Course 6	04	-	01	05
Elective Course 7	04	-	01	05
Elective Course 8	04	-	01	05
Total credits in Elective Courses	20			
Number of Open Elective Courses	One			
	Credits in each Open Elective Course			
Course	Theory	Practical	Tutorial	Credits
Open Elective Course 2	02	-	-	02
Total credits in Open Elective Course	02			
Total credits in Semester IV	**20 + 2 = 22***			

*** Credits from courses offered by the Department of Mathematics**

IV: Course Wise Content Details for M.Sc. Mathematics Programme:

Courses:

Semester I

MMATH18-101: Field Theory
MMATH18-102: Complex Analysis
MMATH18-103: Measure and Integration
MMATH18-104: Differential Equations

Semester II

MMATH18-201: Module Theory
MMATH18-202: Introduction to Topology
MMATH18-203: Functional Analysis
MMATH18-204: Fluid Dynamics

Semester III

MMATH18-301: Any course out of the following

i. Algebraic Topology
ii. Commutative Algebra
iii. Representation of Finite Groups

MMATH18-302: Any course out of the following

i. Fourier Analysis
ii. Matrix Analysis
iii. Theory of Bounded Operators

MMATH18-303: Any course out of the following

i. Advanced Complex Analysis
ii. Advanced Measure Theory
iii. General Topology

MMATH18-304: Any course out of the following

i. Computational Fluid Dynamics
ii. Computational Methods for ODE
iii. Mathematical Programming
iv. Methods of Applied Mathematics

MMATH18-305: (Open Elective) Any course out of the following

i. Coding Theory
ii. Stochastic Calculus for Finance

Semester IV

MMATH18-401: Any course out of the following

i. Advanced Group Theory
ii. Algebraic Number Theory
iii. Simplicial Homology Theory
iv. Theory of Noncommutative rings

MMATH18-402: Any course out of the following

i. Abstract Harmonic Analysis
ii. Frames and Wavelets
iii. Operators on Hardy Hilbert Spaces
iv. Theory of Unbounded Operators

MMATH18-403: Any course out of the following

i. Calculus on $\mathbb{R}^n$
ii. Differential Geometry
iii. Topological Dynamics

MMATH18-404: Any course out of the following

i. Advanced Fluid Dynamics
ii. Computational Methods for PDE
iii. Dynamical Systems
iv. Optimization Techniques & Control Theory

MMATH18-405: (Open Elective) Any course out of the following

i. Cryptography
ii. Support Vector Machines

Course Details:

Semester I

MMATH18-101: Field Theory

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: Fields forms one of the important and fundamental algebraic structures and has an extensive theory dealing mainly with field extensions which arise in the study of roots of polynomials. In this course we study fields in detail with a focus on Galois theory which provides a link between group theory and roots of polynomials.

Course Learning Outcomes: After studying this course the student will be able to

CO1. identify and construct examples of fields, distinguish between algebraic and transcendental extensions, characterize normal extensions in terms of splitting fields and prove the existence of algebraic closure of a field.

CO2. characterize perfect fields using separable extensions, construct examples of automorphism group of a field and Galois extensions as well as prove Artin's theorem and the fundamental theorem of Galois theory.

CO3. classify finite fields using roots of unity and Galois theory and prove that every finite separable extension is simple.

CO4. use Galois theory of equations to prove that a polynomial equation over a field of characteristic is solvable by radicals iff its group (Galois) is a solvable group and hence deduce that a general quintic equation is not solvable by radicals.

Contents:

Unit I: Fields and their extensions, Splitting fields, Normal extensions, Algebraic closure of a field.

Unit II: Separability, Perfect fields, Automorphisms of field extensions, Artin's theorem, Galois extensions, Fundamental theorem of Galois theory.

Unit III: Roots of unity, Cyclotomic polynomials and extensions, Finite fields, Theorem of primitive element and Steinitz's theorem.

Unit IV: Galois theory of equations, Theorem on natural irrationalities, Radical extensions and solvability by radicals.

Suggested Readings:

[1] P.M. Cohn, *Classic Algebra*, John Wiley & Sons Ltd., 2000.

[2] P.M. Cohn, *Basic Algebra: Groups, Rings and Fields,* Springer, 2005.

[3] D.S. Dummit and R.M. Foote, *Abstract Algebra*, Third Edition, Wiley India Pvt. Ltd., 2011.

[4] N. Jacobson, *Basic Algebra,* Volumes I & II, Second Edition, Dover Publications, 2009.

[5] T.W. Hungerford, *Algebra*, Springer-Verlag, 1981.

MMATH 18-102: Complex Analysis

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The course aims to familiarize the learner with complex function theory, analytic functions theory, the concept of index and Cauchy's theorems, integral formulas, singularities and contour integrations and finally provide a glimpse of maximum principle and Schwarz' lemma.

Course Learning Outcomes: After studying this course the student will be able to
CO1. understand analytic function as a mapping on the plane, Mobius transformation and branch of logarithm.
CO2. understand Cauchy's theorems and integral formulas on open subsets of the plane.
CO3. understand the concept of homotopy and homotopic version of Cauchy's theorem and simply connectivity.
CO4. understand how to count the number of zeros of analytic function giving rise to open mapping theorem and Goursat theorem as a converse of Cauchy's theorem.
CO5. know about the kind of singularities of meromorphic functions which helps in residue theory and contour integrations.
CO6. handle integration of meromorphic function with zeros and poles leading to the argument principle and Rouche's theorem.
CO7. know different versions of the maximum principle as well as the Schwarz's lemma representing analytic function on a disk as fractional mappings.

Contents:

Unit I: Analytic functions as mappings, Conformal mapping, Mobius transformations, Branch of logarithm.

Unit II: Power series representation of analytic functions, Maximum modulus theorem, Index of a closed curve, Cauchy's theorem and integral formula on open subset of .

Unit III: Homotopy, Homotopic version of Cauchy's theorem, Simple connectedness, Counting zeros and open mapping theorem, Goursat's theorem, Classification of singularities, Laurent series.

Unit IV: Residue, Contour integration, Argument principle, Rouche's theorem, Maximum principles, Schwarz' lemma.

Suggested Readings:
[1] L.V. Ahlfors, *Complex Analysis*, Mc Graw Hill Co., Indian Edition, 2017.
[2] J.B. Conway, *Functions of One Complex Variable*, Second Edition, Narosa, New Delhi, 1996.
[3] T.W. Gamelin, *Complex Analysis*, Springer, 2001.
[4] L. Hahn, B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, 1996.
[5] D.C. Ullrich, *Complex Made Simple*, American Mathematical Society, 2008.

MMATH18-103: Measure and Integration

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The main objective is to familiarize with the Lebesgue outer measure, Measurable sets, Measurable functions, Integration, Convergence of sequences of functions and their integrals, Functions of bounded variation, L^p-spaces.

Course Learning Outcomes: After studying this course the student will be able to

CO1. verify whether a given subset of $\mathbb{R}$ or a real valued function is measurable.

CO2. understand the requirement and the concept of the Lebesgue integral (a generalization of the Reimann integration) along its properties.

CO3. demonstrate understanding of the statement and proofs of the fundamental integral convergence theorems and their applications.

CO4. know about the concepts of functions of bounded variations and the absolute continuity of functions with their relations.

CO5. extend the concept of outer measure in an abstract space and integration with respect to a measure.

CO6. learn and apply Holder and Minkowski inequalities in L^p-spaces and understand completeness of L^p-spaces and convergence in measures.

Contents:

Unit I: Lebesgue outer measure, Measurable sets, Regularity, Measurable functions, Borel and Lebesgue measurability, Non-measurable sets.

Unit II: Integration of nonnegative functions, General integral, Integration of series, Riemann and Lebesgue integrals.

Unit III: Functions of bounded variation, Lebesgue differentiation theorem, Differentiation and integration, Absolute continuity of functions, Measures and outer measures, Measure spaces, Integration with respect to a measure.

Unit IV: The L^p-spaces, Holder and Minkowski inequalities, Completeness of L^p-spaces, Convergence in measure, Almost uniform convergence, Egorov's theorem.

Suggested Readings:

[1] G. de Barra, *Measure Theory and Integration,* New Age International (P) Ltd., New Delhi, 2014.

[2] M. Capinski and P.E. Kopp, *Measure, Integral and Probability*, Springer, 2005.

[3] E. Hewitt and K. Stromberg, *Real and Abstract Analysis: A Modern Treatment of the Theory of Functions of a Real Variable*, Springer, Berlin, 1975.

[4] H.L. Royden and P.M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.

MMATH 18-104: Differential Equations

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The objective of this course is to study the solutions of first order ODE's, linear second order ODE's, boundary value problems, eigen values and eigen functions of Sturm Lioville systems, stability of systems of ODEs and PDEs.

Course Learning Outcomes: After studying this course the student will be able to

CO1. know about existence, uniqueness and continuity of solutions of first order ODE's, properties of zeros of solutions of linear second order ODE's, boundary value problems.

CO2. understand with eigen values and eigen functions of Sturm–Liouville systems, and the solutions of initial and boundary value problems.

CO3. be well equipped to undertake any advanced course on ordinary as well as partial differential equations.

Contents:

Unit I: Well posed problems, Existence, uniqueness and continuity of solution of ODEs of first order, Picard's method, Existence and uniqueness of solution of simultaneous differential equations of first order and ODEs of higher order, Sturm separation and comparison theorems, Homogeneous linear systems, Non-homogeneous linear systems, Linear systems with constant coefficients.

Unit II: Two point boundary value problems, Green's function, Construction of Green's function, Sturm–Lioville systems, Eigen values and eigen functions, Stability of autonomous system of differential equations, Critical point of an autonomous system and their classification as stable, Asymptotically stable, Strictly stable and unstable, Stability of linear systems with constant coefficients, Linear plane autonomous systems, Perturbed systems, Method of Lyapunov for nonlinear systems.

Unit III: Fourier transform and its application to solution of PDEs, Boundary value problems, Maximum and minimum principles, Uniqueness and continuous dependence on boundary data, Solution of the Dirichlet and Neumann problem for a half plane by Fourier transform method, Solution of Dirichlet problem for a circle in form of Poisson integral formula, Theory of Green's function for Laplace equation in two dimension and application in solution of Dirichlet and Neumann problem for half plane and circle, Theory of Green's function for Laplace equation in three dimension and application in solution of Dirichlet and Neumann problem for semi-infinite spaces and spheres.

Unit IV: Wave equation, Helmholtz's first and second theorems, Green's function for wave equation, Duhamel's principles for wave equation, Diffusion equation, Solution of initial boundary value problems for diffusion equation, Green's function for diffusion equation, Duhamel's principles for heat equation.

Suggested Readings:

[1] E.A. Coddington, *An Introduction to Ordinary Differential Equations*, Dover Publications, 2012.
[2] T. Myint-U, *Ordinary Differential Equations*, Elsevier, North-Holland, 1978.
[3] S.L. Ross, *Differential Equation*s, Second Edition, John Wiley & Sons, India, 2007.
[4] I.N. Sneddon, *Elements of Partial Differential Equations*, Dover Publications, 2006.

Semester II

MMATH18-201: Module Theory

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: In this course a new algebraic structure, namely, modules is introduced and studied in detail. Modules are the generalization of vector spaces when the underlying field is replaced by an arbitrary ring. The study of modules over a ring also provides an insight into the structure of ring.

Course Learning Outcomes: After studying this course the student will be able to

CO1. Identify and construct example of modules, and apply homomorphism theorems on the same.

CO2. distinguish between projective, injective, free, and semi simple modules.

CO3. prove universal property of tensor product of modules, Hilbert basis theorem.

CO4. define and characterize Notherian, Artinian module, and apply the structure theorem of finitely generated modules over PID.

Contents:

Unit I: Basic concepts of module theory, direct product and direct sum of modules, exact sequences, split exact sequences.

Unit II: Categories and functors, free modules, projective and injective modules, dual basis lemma, Baer's criterion, divisible modules.

Unit III: Tensor product of modules, chain conditions, Hilbert basis theorem.

Unit IV: Modules over PID's, semi simple modules.

Suggested Readings:

[1] M.F. Atiyah and I.G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.

[2] P.M. Cohn, *Classic Algebra*, John Wiley & Sons Ltd., 2000.

[3] P.M. Cohn, *Basic Algebra: Groups, Rings and Fields,* Springer, 2005.

[4] D.S. Dummit and R.M. Foote, *Abstract Algebra*, Third Edition, Wiley India Pvt. Ltd., 2011.

[5] T.W. Hungerford, *Algebra*, Springer-Verlag, 1981.

[6] N. Jacobson, *Basic Algebra,* Volumes I & II, Second Edition, Dover Publications, 2009.

MMATH18-202: Introduction to Topology

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: To introduce basic concepts of point set topology, basis and subbasis for a topology and order topology. Further, to study continuity, homeomorphisms, open and closed maps, product and box topologies and introduce notions of connectedness, path connectedness, local connectedness, local path connectedness, convergence, nets, countability axioms and compactness of spaces.

Course Learning Outcomes: After studying this course the student will be able to

CO1. determine interior, closure, boundary, limit points of subsets and basis and subbasis of topological spaces.

CO2. check whether a collection of subsets is a basis for a given topological spaces or not, and determine the topology generated by a given basis.

CO3. identify the continuous maps between two spaces and maps from a space into product space and determine common topological property of given two spaces.

CO4. determine the connectedness and path connectedness of the product of an arbitrary family of spaces.

CO5. find Hausdorff spaces using the concept of net in topological spaces and learn about 1^{st} and 2^{nd} countable spaces, separable and Lindelöf spaces.

CO6. learn Bolzano−Weierstrass property of a space and prove Tychonoff theorem.

Contents:

Unit I: Topological spaces, Derived concepts, Interior, closure, boundary and limit points of subsets.

Unit II: Basis and subbasis for a topology, Order topology, Subspaces, Continuous functions, Homeomorphism, Product topology, Metrizability of products of metric spaces.

Unit III: Connected spaces, Components, Path connected spaces, Local connectedness, Local path-connectedness, Convergence, Sequences and nets, Hausdorff spaces.

Unit IV: 1^{st} and 2^{nd} countable spaces, Separable and Lindelöf spaces, Compactness, Tychonoff's theorem, Bolzano−Weierstrass property, Countable compactness.

Suggested Readings:

[1] G.E. Bredon, *Topology and Geometry*, Springer, 2014.

[2] J. Dugundji, *Topology*, Allyn and Bacon Inc., Boston, 1978.

[3] J.L. Kelley, *General Topology*, Dover Publications, 2017.

[4] J.R. Munkres, *Topology*, Second Edition, Pearson, 2015.

[5] T.B. Singh, *Elements of Topology*, CRC Press, Taylor & Francis, 2013.

[6] S. Willard, *General Topology*, Dover Publications, 2004.

MMATH18-203: Functional Analysis

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: To familiarize with the basic tools of Functional Analysis involving normed spaces, Banach spaces and Hilbert spaces, their properties dependent on the dimension and the bounded linear operators from one space to another**.**

Course Learning Outcomes: After studying this course the student will be able to

CO1. verify the requirements of a norm, completeness with respect to a norm, relation between compactness and dimension of a space, check boundedness of a linear operator and relate to continuity, convergence of operators by using a suitable norm, compute the dual spaces.

CO2. distinguish between Banach spaces and Hilbert spaces, decompose a Hilbert space in terms of orthogonal complements, check totality of orthonormal sets and sequences, represent a bounded linear functional in terms of inner product, classify operators into self-adjoint, unitary and normal operators.

CO3. extend a linear functional under suitable conditions, compute adjoint of operators, check reflexivity of a space, ability to apply uniform boundedness theorem, open mapping theorem and closed graph theorem, check the convergence of operators and functional and weak and strong convergence of sequences.

CO4. compute the spectrum of operators and classify the set into subclasses, show the spectrum to be nonempty, give expansion of resolvent operator.

Contents:

Unit I: Normed spaces, Banach spaces, Finite dimensional normed spaces and subspaces, Compactness and finite dimension, Bounded and continuous linear operators, Linear operators and functionals on finite dimensional spaces, Normed spaces of operators, Dual spaces.

Unit II: Hilbert spaces, Orthogonal complements and direct sums, Bessel's inequality, Total orthonormal sets and sequences, Representation of functionals on Hilbert spaces, Hilbert adjoint operators, Self-adjoint, unitary and normal operators.

Unit III: Hahn Banach theorems for real and complex normed spaces, Adjoint operator, Reflexive spaces, Uniform boundedness theorem strong and weak convergence, Convergence of sequences of operators and functionals, Open mapping theorem, Closed graph theorem.

Unit IV: Spectrum of an operator, Spectral properties of bounded linear operators, Non-emptiness of the spectrum.

Suggested Readings:

[1] G. Bachman and L. Narici, *Functional Analysis*, Dover Publications, 2000.

[2] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, India, 2009.

[3] E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, India, 2006.

[4] M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.

MMATH18-204: Fluid Dynamics

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: Prepare a foundation to understand the motion of fluid and develop concept, models and techniques which enables to solve the problems of fluid flow and help in advanced studies and research in the broad area of fluid motion.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand the concept of fluid and their classification, models and approaches to study the fluid flow.

CO2. formulate mass and momentum conservation principle and obtain solution for non-viscous flow.

CO3. know potential theorems, minimum energy theorem and circulation theorem.

CO4. understand two dimensional motion, circle theorem and Blasius theorem.

CO5. understand three dimensional motions, Weiss's and Butler's sphere theorems and Kelvin's inversion theorem.

CO7. understand the concept of stress and strain in viscous flow and to derive Navier–Stokes equation of motion and solve some exactly solvable problems.

Contents:

Unit I: Classification of fluids, Continuum model, Eulerian and Lagrangian approach of description, Differentiation following the fluid motion, Irrotational flow, Vorticity vector, Equipotential surfaces, Streamlines, pathlines and streak lines of particles, Stream tube and stream surface, Mass flux density, Conservation of mass leading to equation of continuity (Euler's form), Boundary surface, Conservation of momentum and its mathematical formulation (Euler's form), Integration of Euler's equation under different conditions, Bernoulli's equation, steady motion under conservative body forces.

Unit II: Theory of irrotational motion, Kelvin's minimum energy and circulation theorems, Potential theorems, Two-dimensional flows of irrotational, incompressible fluids, Complex potential, Sources, sinks, doublets and vortices, Milne–Thomson circle theorem, Images with respect to a plane and circles, Blasius theorem.

Unit III: Three-dimensional flows, Sources, sinks, doublets, Axi-symmetric flow and Stokes stream function, Butler sphere theorem, Kelvin's inversion theorem, Weiss's sphere theorem, Images with respect to a plane and sphere, Axi-symmetric flows and stream function, Motion of cylinders and spheres.

Unit IV: Viscous flow, stress and strain analysis, Stokes hypothesis, Navier–Stokes equations of motion, Some exactly solvable problems in viscous flows, Steady flow between parallel plates, Poiseuille flow, Steady flow between concentric rotating cylinders.

Suggested Readings:

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.

[2] R.W. Fox, P.J. Pritchard and A.T. McDonald, *Introduction to Fluid Mechanics*, Seventh Edition, John Wiley & Sons, 2009.

[3] P.K. Kundu, I.M. Cohen, D.R. Dowling, *Fluid Mechanics*, Sixth Edition, Academic Press, 2016.

Semester III

MMATH18-301(i): Algebraic Topology

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: Introduce the notion of homotopy, groups with pointed spaces and covering spaces which is closely associated with the fundamental groups. Moreover, to introduce the concept of free groups and presentation of a group and to compute the fundamental group of wedge of circles, Klein bottle, adjunction of a disc and a path connected space.

Course Learning Outcomes: After completing this course a student will be able to
CO1. grasp the basics of Algebraic Topology.
CO2. determine fundamental groups of some standard spaces like Euclidean spaces and spheres.
CO3. understand proofs of some beautiful results such as Fundamental theorem of Algebra, Brower's fixed-point theorem, Borsuk–Ulam theorem.

Contents:

Unit I: Homotopic maps, homotopy type, retract and deformation retract.

Unit II: Fundamental group, Calculation of fundamental groups of n-sphere, cylinder, torus, and punctured plane, Brouwer's fixed-point theorem, Fundamental theorem of Algebra.

Unit III: Free products, Free groups, Seifert–Van Kampen theorem and its applications.

Unit IV: Covering projections, Lifting theorems, Relations with the fundamental group, Universal covering space, Borsuk–Ulam theorem, Classification of covering spaces.

Suggested Readings:
[1] G.E. Bredon, *Geometry and Topology*, Springer, 2014.
[2] W.S. Massey, *A Basic Course in Algebraic Topology*, World Publishing Corporation, 2009.
[3] J.J. Rotman, *An Introduction to Algebraic Topology*, Springer, 2011.
[4] T.B. Singh, *Elements of Topology*, CRC Press, Taylor & Francis, 2013.
[5] E.H. Spanier, *Algebraic Topology*, Springer-Verlag, 1989.

MMATH18-301(ii): Commutative Algebra

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: To study commutative rings with unity and modules over the same that helps in developing basic foundation in other areas of mathematics such as algebraic geometry, homological algebra and algebraic number theory.

Course Learning Outcomes: After studying this course the student will be able to

CO1. know the localization of rings at a prime ideal that is an algebraic analogue of the geometric notion concentrating attention near a point.

CO2. know more closely the polynomial rings, power series rings in one or more variables over a commutative ring and their prime spectrum.

CO3. define, identify, and elaborate integral closure of rings, valuations rings, discrete valuation rings, structure theorem of Artin rings.

Contents:

Unit I: Extension and contraction of ideals, Prime spectrum of rings, Jacobson radical of a ring, Prime avoidance lemma, Rings of formal power series, Restriction and extension of scalars.

Unit II: Localisation, Local properties, Extended and contracted ideals in rings of fractions, Primary decomposition, First and second uniqueness theorem of primary decomposition.

Unit III: Integral dependence, Going up theorem, Going down theorem, Integrally closed domains, Valuation rings, Hilbert's Nullstellensatz theorem.

Unit IV: Noetherian rings, Primary decomposition in Noetherian rings, Artin rings, Structure theorem for Artin rings, Discrete valuation rings, Dedekind domains, Fractional ideals.

Suggested Readings:

[1] M.F. Atiyah and I.G. MacDonald, *Introduction to Commutative Algebra*, CRC Press, Taylor & Francis, 2018.

[2] B. Singh, *Basic Commutative Algebra*, World Scientific, 2011.

[3] D. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer, 2004.

[4] O. Zariski and P. Samuel, *Commutative Algebra*, Volume I & II, Springer, 1975.

[5] R.Y. Sharp, *Steps in Commutative Algebra*, Cambridge University Press, 2000.

MMATH18-301(iii): Representation of Finite Groups

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: To provide a complete description of an abstract finite group by representing it in terms of group of matrices and practice their applications and interpretations in a range of situations.

Course Learning Outcomes: After studying this course the student will be able to
CO1. learn the ways of writing a group as group of matrices.
CO2. know about Schur's lemma and tensor products.
CO3. compute character tables of symmetric groups and alternating groups.
CO4. know the range of applications of the theory that extends beyond the boundaries of pure mathematics and includes theoretical physics, quantum mechanics.

Contents:

Unit I: Representation of Groups, Equivalent representations, Trivial and faithful representations, *FG*-modules and submodules, Permutation modules for *G* over *F*, *FG*-modules and equivalent representations, Reducible and irreducible *FG*-modules, Group Algebra of *G*, Regular *FG*-module and regular representations, *FG*-homomorphisms, Direct sums of *FG*-modules, Maschke's theorem for *FG*-modules and matrices, Completely reducible matrix representations and *FG*-modules, *CG*-homomorphisms.

Unit II: Schur's lemma and its converse, Representations of finite abelian groups, applications of Schur's lemma, Irreducible modules and group algebra, Space of *CG*-homomorphisms, Elementary properties of group characters, Faithful characters, Irreducible characters, Regular characters, Permutation characters, Inner products of characters, Decomposing *CG*-modules into direct sum of *CG*-submodules, Number of irreducible characters.

Unit III: Row and column orthogonality relations, Lifted characters and normal subgroups, Linear characters, Character tables of symmetric and alternating groups of degree 4 and 5, Tensor products, Restriction and induced modules and characters, Frobenius reciprocity theorem.

Unit IV: Algebraic integers and their properties, Characters of groups of order *pq* including a class of Frobenius groups, Character of some *p*-groups, Burnsides's (*p*, *q*)-theorem.

Suggested Readings:
[1] G. James and M. Liebeck, *Representations and Character of Groups*, Second Edition, Cambridge University Press, 2005.
[2] C.W. Curtis and I. Reiner, *Representation Theory of Finite Groups and Associative Algebras*, American Mathematical Society, 2006
[3] W. Fulton and J. Harris, *Representation Theory, A First Course*, Springer-Verlag, 2004.
[4] I.M. Issacs, *Character Theory of Finite Groups*, American Mathematical Society reprint, 2006.
[5] W. Ledermann, *Introduction to Group Characters*, Cambridge University Press, Cambridge, Second Edition, 1987.

MMATH18-302(i): Fourier Analysis

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: To provide an understanding of Fourier series, their convergence and Fourier transform and inverse Fourier transforms, and practice their application and interpretation in a range of situations.

Course Learning Outcomes: After studying this course the student will be able to

CO1. derive a Fourier series of a given periodic function by evaluating Fourier coefficients.

CO2. obtain the complex exponential Fourier series of a function and relate complex Fourier coefficients to the Fourier cosine and sine coefficients and also to know if a Fourier series can be constructed to represent an arbitrary function.

CO3. calculate the Fourier transform or inverse transform of common functions including rectangular, Gaussian, delta, unit-step, sinusoidal and exponential decays.

CO4. calculate the Fourier transform of periodic functions including the cosine, sine and Dirac comb functions.

CO5. understand basic topological groups, Haar measure, Fourier transform, inverse Fourier transform, Plancherel formula.

Contents:

Unit I: Convergence and divergence of Fourier series, Fejer's theorem, Approximate indentities, Classical kernels, Fejer's, Poisson's and Dirichlet's summability in norm and pointwise summability.

Unit II: Fatou's theorem, Inequalities of Hausdorff and Young, Examples of conjugate function series, Fourier tranform, Kernels on $\mathbb{R}$.

Unit III: Basic properties of topological groups, separation properties, subgroups, quotient groups and connected groups, Notion of Haar measure on topological groups with emphasis on $\mathbb{R}$, $\mathbb{T}$ and $\mathbb{Z}$ and some simple matrix groups,

Unit IV: $L^1(G)$ and convolution with special emphasis on $L^1(\mathbb{R})$, $L^1(\mathbb{T})$ and $L^1(\mathbb{Z})$. Plancherel theorem on abelian groups, Plancherel measure on $\mathbb{R}$, $\mathbb{T}$ and $\mathbb{Z}$, maximal ideal space of $L^1(G)$ (G an abelian topological group).

Suggested Readings:

[1] Y. Katznelson, *An Introduction to Harmonic Analysis*, Third Edition, Cambridge University Press, 2004.

[2] H. Helson, *Harmonic Analysis*, Hindustan Book Agency, 2010.

[3] E. Hewitt and K.A. Ross, *Abstract Harmonic Analysis*, Second Edition, Volume I, Springer Verlag, 1994.

MMATH18-302(ii): Matrix Analysis

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The main objective of this course is to introduce certain topics of matrix analysis from the point of view of functional analysis. It will equip the students with several tools and ideas which can be used in variety of applications.

Course Learning Outcomes: After studying this course the student will be able to
CO1. know about the notions of compactness and connectedness in general linear group.
CO2. learn about analytic and geometric properties of vector norms.
CO3. know about location and perturbation of eigenvalues.
CO4. Learn about positive definite matrices and positive semi-definite ordering.
CO5. learn about majorization and doubly stochastic matrices.

Contents:

Unit I: Closed subgroups of general linear group, Examples and their compactness and connectedness, Matrix exponential.

Unit II: Norm for vectors and matrices, Analytic properties of vector norms, Geometric properties of vector norms, Matrix norms, Error in inverses and solution of linear systems.

Unit III: Location and perturbation of eigenvalues, Geršgorin discs, Other inclusion regions, Positive definite matrices.

Unit IV: Polar form and singular value decomposition, Schur product theorem, Positive semi-definite ordering, Inequalities for positive definite matrices, Majorisation and doubly stochastic matrices.

Suggested Readings:
[1] R. Bhatia, *Matrix Analysis*, Springer, 1997.
[2] B.C. Hall, *Lie Groups, Lie Algebras, and Representations: An Elementary Introduction*, Second Edition, Springer, 2015.
[3] R.A. Horn and C.R. Johnson, *Matrix Analysis*, Cambridge University Press, 2012.
[4] C.D. Meyer, *Matrix Analysis and Applied Linear Algebra*, SIAM, 2000.
[5] F. Zhang, *Matrix Theory: Basic Results and Techniques*, Springer Verlag, 1999.

MMATH18-302(iii): Theory of Bounded Operators

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The course aims to introduce some types of bounded linear operators and spectrum theory which play central role in both pure and applied mathematics.

Course Learning Outcomes: After studying this course the student will be able to

CO1. identify spectrum, particularly point spectrum and resolvent of standard operators like shifts and multiplication and to understand the spectral theorem for bounded linear operators.

CO2. understand the basic properties of bounded linear operators on normed, Banach and Hilbert spaces and apply these properties to solve simple problems.

CO3. understand the concepts of compactness, self-adjointness and positivity of bounded linear operators.

CO4. understand trace class and Hilbert−Schmidt operators.

Contents:

Unit I: Spectrum of a bounded operator, Review of basic concepts, Point, continuous and residue spectrum, Notions of uniform, strong and weak operator convergence on the space of bounded linear operators, Approximate point spectrum and compression spectrum, Spectral mapping theorem for polynomials.

Unit II: Compact linear operators, Basic properties, adjoint of compact operators, Spectral properties of compact operators, Fredholm alternative.

Unit III: Spectral theory of self-adjoint operators, Spectral properties of self-adjoint operators, Positive operators and their properties, Spectral representation of a self-adjoint compact operator, Spectral family of a self-adjoint operator and its properties, Spectral representation of a self-adjoint operator, Continuous functions of self-adjoint operators.

Unit IV: Polar decomposition, Singular values, Trace class operators, Trace norm and Hilbert Schmidt operators.

Suggested Readings:

[1] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency, India, 2009.

[2] J.B. Conway, *A Course in Operator Theory*, American Mathematical Society, 2000.

[3] E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, India, 2006.

[4] M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.

MMATH 14-303(i): Advanced Complex Analysis

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The primary objective of this course is to understand the notion of logarithmically convex function and its fusion with maximum modulus theorem, the spaces of continuous, analytic and meromorphic functions, Runge's theorem and topics related with it, introduce harmonic function theory leading to Dirichlet's problem, theory of range of an entire function leading to Picard and related theorems.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand the basics of logarithmically convex function that helps in extending maximum modulus theorem.

CO2. be familiar with metric on spaces of analytic, meromorphic and analytic functions, equi-continuity and normal families leading to Arzela–Ascoli and related theorems.

CO3. appreciate the richness of simply connected region which connects various fields-topology, analysis and algebra.

CO4. know harmonic function theory on a disk and how it helps in solving Dirichlet's problem and the notion of Green's function.

CO5. know how big the range of an entire function is as well as Picard and related theorems.

Contents:

Unit I: Hadamard's three circles theorem, Phragmen–Lindelöf theorem, Spaces of continuous functions, Spaces of analytic functions, Hurwitz's theorem, Montel's theorem, Spaces of meromorphic functions.

Unit II: Riemann mapping theorem, Weierstrass' factorization theorem, Factorization of sine function, Runge's theorem, Simply connected regions, Mittag–Leffler's theorem.

Unit III: Harmonic functions, Maximum and minimum principles, Harmonic function on a disk, Harnack's theorem, Sub-harmonic and super-harmonic functions, Maximum and minimum principles, Dirichlet's problems, Green's function.

Unit IV: Entire functions, Jensen's formula, Bloch's theorem, Picard theorem, Schottky's theorem.

Suggested Readings:

[1] L.V. Ahlfors, *Complex Analysis*, Mc. Graw Hill Co., Indian Edition, 2017.
[2] J.B. Conway, *Functions of One Complex Variable*, Second Edition, Narosa, New Delhi, 1996.
[3] L. Hahn, B. Epstein, *Classical Complex Analysis*, Jones and Bartlett, 1996.
[4] W. Rudin, *Real and Complex Analysis*, Third Edition, Tata McGraw-Hill, 2006.
[5] D.C. Ullrich, *Complex Made Simple*, American Mathematical Society, 2008.

MMATH18-303(ii): Advanced Measure Theory

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: Introduce negative real valued and complex valued measures, decomposition of the measure space and the measure, extension of a measure, integral representation of measures and functionals, product measure and uniqueness of Lebesgue measure in Euclidean space.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand signed measures and complex measures, ability to use Hahn decomposition, Jordan decomposition, Radon–Nikodym theorem and recognize singularity of measures.

CO2. verify conditions under which a measure defined on a semi-algebra or algebra is extendable to a sigma-algebra and to get the extended measure, and to prove the uniqueness up to multiplication by a scalar of Lebesgue measure in $\mathbb{R}^n$ as a translation invariant Borel measure.

CO3. learn and apply Riesz representation theorem for a bounded linear functional on L^p spaces, understand product measure and the results of Fubini and Tonelli.

CO4. to understand the concepts of Baire sets, Baire measures, regularity of measures on locally compact spaces, Riesz–Markov representation theorem related to the representation of a bounded linear functional on the space of continuous functions.

Contents:

Unit I: Signed measures, Hahn and Jordan decomposition theorems, Mutually singular measures, Radon–Nikodym theorem, Lebesgue decomposition.

Unit II: Caratheodory extension theorem, Lebesgue measure on $\mathbb{R}^n$, Uniqueness up to multiplication by a scalar of Lebesgue measure in $\mathbb{R}^n$ as a translation invariant Borel measure.

Unit III: Riesz representation theorem for bounded linear functionals on L^p -spaces, Product measures, Fubini's theorem, Tonelli's theorem.

Unit IV: Baire sets, Baire measures, Continuous functions with compact support, regularity of measures on locally compact spaces, Regularity of Lebesgue measure in $\mathbb{R}^n$, Riesz–Markov representation theorem.

Suggested Readings:

[1] C.D. Aliprantis and O. Burkinshaw, *Principles of Real Analysis*, Academic Press, Indian Reprint, 2011.

[2] A.K. Berberian, *Measure and Integration*, AMS Chelsea Publications, 2011.

[3] P.R. Halmos, *Measure Theory*, Springer, 2014.

[4] M.E. Taylor, *Measure Theory and Integration*, American Mathematical Society, 2006.

[5] H.L. Royden and P.M. Fitzpatrick, *Real Analysis*, Fourth Edition, Pearson, 2015.

MMATH18-303(iii): General Topology

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: It is a second course in Topology with main objective to teach students many important results on several useful topics including quotient spaces, local compactness, one point compactification, separation axioms, Urysohn lemma, Tietze extension theorem, paracompactness, metrization theorems and partition of unity. In addition, the course aims to provide students the awareness of tools for carrying out advanced research later in Topology and related areas.

Course Learning Outcomes: After studying this course the student will be able to
CO1. know about nice examples of quotient spaces including cones and suspensions.
CO2. find one point compactification of spaces like real line and n-sphere
CO3. know interesting results on complete regularity and Stone–Cech compactification.
CO4. have studied celebrated results like Urysohn lemma, Tietze extension theorem.
CO5. know about useful Urysohn metrization theorem and Nagata Smirnov metrization theorem.
CO6. know characterizations of paracompactness in regular spaces and partition of unity.

Contents:

Unit I: Quotient spaces, Identification maps, cones, suspensions, local compactness and one-point compactification.

Unit II: Proper maps, Regularity, Complete regularity, Stone–Cech compactification.

Unit III: Normality, Urysohn lemma, Tietze extension theorem, Urysohn metrization theorem.

Unit IV: Nagata–Smirnov metrization theorem, Paracompactness, Characterizations of paracompactness in regular spaces, Partition of unity.

Suggested Readings:
[1] J. Dugundji, *Topology*, Allyn and Bacon Inc., Boston, 1978.
[2] R. Engelking, *General Topology*, Heldermann Verlag, 1989.
[3] J.L. Kelley, *General Topology*, Dover Publications, 2017.
[4] J.R. Munkres, *Topology*, Second Edition, Pearson, 2015.
[5] T.B. Singh, *Elements of Topology*, CRC Press, Taylor & Francis, 2013.
[6] S. Willard, *General Topology*, Dover Publications, 2004.

MMATH18-304(i): Computational Fluid Dynamics

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The foremost objective of this course is to introduce the students some real world applications of computational fluid dynamics. This course aims at students to know the basic conservation principles of mass, momentum, energy, discretization of governing equations and thereby computing the numerical solutions using of the flow variables using finite difference and finite volume methods.

Course Learning Outcomes: After studying this course the student will be able to

CO1. know the basic conservation principles of mass, momentum and energy and their governing equations.

CO2. understand the basic aspects of discretization and numerical solutions using both finite difference and finite volume methods.

CO3. know some popular algorithms like SIMPLE and SIMPLER used to obtain the solutions of steady and unsteady flow problems by finite volume methods.

Contents:

Unit I: Basics of discretization using finite differences, Single and multi step schemes for parabolic and hyperbolic PDEs, Finite difference schemes for convection-diffusion equation, Accuracy, Consistency, Stability and Convergence of a finite difference scheme, Courant Friedrich Lewy condition, Von Neumann and matrix stability analysis of finite difference schemes, Methods for solving discretized equations.

Unit II: Mathematical description of physical phenomenas, Finite volume method for diffusion and convection-diffusion equations, Discretization of one and two-dimensional steady state diffusion and convection-diffusion equations, Central difference, upwind, exponential, hybrid, power-law and QUICK schemes and their properties.

Unit III: Flow field calculation, pressure-velocity coupling, vorticity-stream function approach, primitive variables, staggered grid, pressure and velocity corrections, pressure correction equation, SIMPLE, SIMPLER and PISO algorithms.

Unit IV: Finite volume methods for unsteady flows, Discretization of one-dimensional transient heat conduction, explicit, fully implicit and Crank–Nicolson schemes, Implementation of boundary conditions.

Suggested Readings:

[1] R.H. Pletcher, J.C. Tannehill and D.A. Anderson, *Computational Fluid Mechanics and Heat Transfer*, CRC Press, Taylor and Francis, 2013.

[2] J.D. Anderson, *Computational Fluid Dynamics*, McGraw-Hill, 1995.

[3] S.V. Patankar, *Numerical Heat Transfer and Fluid Flow*, CRC Press, Taylor and Francis, Indian Edition, 2017.

[4] J.C. Strikwerda, *Finite Difference Schemes and Partial Differential Equations*, Second Edition, SIAM, 2004.

[5] J.W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 2013.

[6] H.K. Versteeg, and W. Malalasekera, *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, Second Edition, Pearson, 2008.

MMATH18-304(ii): Computational Methods for ODEs

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The aim of this course is to enable students to design and analyze numerical methods to approximate solutions to differential equations for which finding an analytic (closed-form) solution is not possible. This course is devoted to learning basic scientific computing for solving differential equations. The concept and techniques included in this course enable the student to construct and use elementary MATLAB, MATHEMATICA programs for differential equations.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand the key ideas, concepts and definitions of the computational algorithms, origins of errors, convergence theorems.

CO2. decide the best numerical method to apply to solve a given differential equation and quantify the error in the numerical (approximate) solution.

CO3. analyze an algorithm's accuracy, efficiency and convergence properties.

Contents:

Unit I: Initial Value Problems (IVPs) for the system of ODEs, Difference equations, Numerical Methods, Local truncation errors, Global truncation error, Stability analysis, Interval of absolute stability, Convergence and consistency.

Unit II: Single-step methods, Taylor series method, Explicit and implicit Runge–Kutta methods and their stability and convergence analysis, Extrapolation method, Runge–Kutta method for the second order ODEs and Stiff-system of differential equations.

Unit III: Multi-step methods, Explicit and implicit multi-step methods, General linear multi-step methods and their stability and convergence analysis, Adams–Moulton method, Adams–Bashforth method, Nystorm method, Multistep methods for second order IVPs.

Unit IV: Boundary Value Problems (BVPs), Two point non-linear BVPs for second order ordinary differential equations, Finite difference methods, Convergence analysis, Difference scheme based on quadrature formula, Difference schemes for linear eigen value problems, Mixed boundary conditions, Finite element methods, Assemble of element equations, Variational formulation of BVPs and their solutions, Galerikin method, Ritz method, Finite element solution of BVPs.

Note: Use of non-programmable scientific calculator is allowed in theory examination.

Suggested Readings:

[1] K.E. Atkinson, W. Han and D.E. Stewart, *Numerical Solution of Ordinary Differential Equations*, John Wiley & Sons, 2009.

[2] J.C. Butcher, *Numerical Methods for Ordinary Differential Equations*, Third Edition, John Wiley & Sons, 2016.

[3] J.D. Lambert, *Numerical Methods for Ordinary Differential Systems: The Initial Value Problem*, John Wiley & Sons, 1991.

MMATH18-304(iii): Mathematical Programming

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The objective of this course is to study optimality conditions, Lagrangian duality and numerical methods of mathematical programming problems with nonlinear objective and nonlinear constraints.

Course Learning Outcomes: After studying this course the student will be able to

CO1. derive first and second order optimality conditions for a nonlinear programming problem and consider convex functions for deriving sufficient optimality conditions.

CO2. understand duality theory in terms of Lagrangian function and investigate saddle point theory.

CO3. understand numerical methods like Wolfe's method, convex simplex method and penalty function methods for solving different types of nonlinear programming problems.

Contents:

Unit I: Existence theorems, First order optimality conditions and second order optimality conditions for unconstrained optimization problems.

Unit II: Convex functions, Differentiable convex functions, Optimization on convex sets, Separation theorems, Fritz John optimality conditions for constrained nonlinear programming problems, Constraint qualifications, Karush−Kuhn−Tucker conditions in nonlinear programming, Second order conditions in nonlinear programming

Unit III: Langragian saddle points, Duality in nonlinear programming, Strong duality in convex programming, duality for linear and quadratic problems.

Unit IV: Quadratic programming, Wolfe's method as application of Karush−Kuhn−Tucker conditions, convex simplex method, Penalty function methods.

Suggested Readings:

[1] M.S. Bazaraa, H.D. Sherali and C.M. Shetty, *Nonlinear Programming: Theory and Algorithms*, John Wiley & Sons, 2013.

[2] O. Güler, *Foundations of Optimization*, Springer 2010.

[3] D.G. Luenberger and Y. Ye, *Linear and Nonlinear Programming*, Springer, 2008.

MMATH18-304 (iv): Methods of Applied Mathematics

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The main objectives of the course are to familiarize the learner with Dimensional Analysis, Scaling, Perturbation methods, WKB approximations, methods for solving integral equations and variational problems.

Course Learning Outcomes: After studying this course the student will be able to

CO1. know the concept of dimensional analysis and learn the main applications of dimensional analysis.

CO2. learn the applications of Buckingham Pi theorem, scaling, perturbation methods and WKB approximations in detail.

CO3. compute solutions to Volterra integral equations by method of resolvent kernel, method of successive approximations, method of Laplace transform, system of Volterra integral equations and integro-differential equation.

CO4. determine the solutions of Fredholm integral equations and derivation of Hilbert–Schmidt theorem.

CO5. understand the formulation of variational problems, the variation of a functional and its properties, extremum of functional, necessary condition for an extremum.

Contents:

Unit I: Dimensional analysis, Buckingham Pi Theorem, Scaling, Perturbation methods, Regular perturbations, Singular perturbations, WKB approximations.

Unit II: Integral equation, Introduction and relation with linear differential equation. Volterra integral equations and its solutions, Method of resolvent kernel, Method of successive approximations, Convolution type of equation, Method of Laplace Transform, system of Volterra integral equations, Integro-differential equation, Abel's integral equation and its generalizations.

Unit III: Fredholm integral equations and its solutions, Method of resolvent kernels, Method of successive approximations, Integral equations with degenerate kernels, Eigen values and eigen functions and their properties, Hilbert Schmidt theorem, Non homogeneous Fredholm integral equation with symmetric kernel, Fredholm alternative.

Unit IV: Variational problems, the variation of a functional and its properties, Extremum of functional, Necessary condition for an extremum, Euler's equation and its generalization, Variational derivative, General variation of a functional and variable end point problem, Sufficient conditions for the extremum of a functional.

Suggested Readings:

[1] M. Gelfand and S.V. Fomin, *Calculus of Variations*, Prentice Hall, Inc., 2000.
[2] F.B. Hildebrand, *Methods of Applied Mathematics*, Dover Publications, 1992.
[3] M.L. Krasnov, *Problems and Exercises Integral Equations*, Mir Publication Moscow, 1971.
[4] D. Logan, *Applied Mathematics: A Contemporary Approach*, John Wiley & Sons, 1997.

MMATH18-305(i): Coding Theory

Total Marks: 50 (35 End Semester Examination + 15 Internal Assessment)

Course Objectives: This course provides an introduction to algebraic coding theory, particularly linear codes. Bounds on the parameters have been discussed and cyclic codes have been explored. Some well-known codes such as BCH and Reed Muller codes have been described.

Course Learning Outcomes: After studying this course the student will be able to
CO1. get an insight into matrix representation of a code as well as encoding and decoding.
CO2. understand Hamming codes, MDS codes and Reed–Muller codes.
CO3. learn about cyclic codes and their generator polynomial.

Contents:

Unit I: Introduction to algebraic coding theory, Linear codes, Hamming weight, Generator matrix, Parity check matrix, Equivalence of linear codes, Bounds on codes, Hamming codes, MDS codes, Propagation rules, Lengthening of code, Subcodes, Puncturing of code, Direct sum construction, Reed–Muller codes, Subfield codes.

Unit II: Cyclic codes, Cyclic codes as ideals, Generator polynomial of cyclic codes, Matrix representation of cyclic codes**,** Burst error correcting codes, Some special cyclic codes, BCH codes, Reed Solomon codes.

Suggested Readings:
[1] S. Ling and C. Xing, *Coding Theory: A First Course*, Cambridge University Press, 2004
[2] R. Hill, *A First Course in Coding Theory*, Oxford University Press, 1986.
[3] W.C. Huffman and V. Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press, 2010.

MMATH18-305(ii): Stochastic Calculus for Finance

Total Marks: 50 (35 End Semester Examination + 15 Internal Assessment)

Course Objectives: This course is an introduction to stochastic calculus and its applications to financial mathematics. The main aim is to familiarise the Ito integral, Black−Scholes theory for pricing and hedging of financial derivatives and risk neutral valuation.

Course Learning Outcomes: After studying this course the student will be able to

CO1 define a probability measure and probability space, define a random variable and give examples, understand expectation as an integral with respect to probability measure, define conditional expectation and define a martingale.

CO2 define a Brownian motion and find its quadratic variation, define an Ito integral, define an Ito process, write Ito's formula for Brownian motion and for Ito process, use Ito's formula for solving SDE's e.g. GBM and OU process.

CO3 understand the concept of options, derive the Black−Scholes PDE and to write its solution, derive the formulae for option Greeks delta, gamma, theta, rho and vega.

CO4 use Girsanov's theorem for change of measure, use the risk neutral valuation formula for pricing derivatives, derive Black−Scholes formulae for the price of call and put options, understand the first and second fundamental theorem of asset pricing.

Contents:

Unit I: Sigma algebra, Probability measure, Random variable, Expectation, Dependence, Conditional expectation, Filtration, Martingale, Brownian Motion, Quadratic variation of Brownian motion, Ito integral, Ito's formula for Brownian motion, Ito's formula for Ito process.

Unit II: Financial derivatives, Black−Scholes PDE and its solution, Greeks, Put-call parity, Girsanov's theorem, Pricing under risk neutral measure, Black-Scholes formulae, Fundamental theorems of asset pricing, Forwards and futures.

Suggested Readings:

[1] M. Baxter and A. Rennie, *Financial Calculus: An Introduction to Derivative Pricing,* Cambridge University Press, 1996.

[2] N.H. Bingham and R. Kiesel, *Risk-Neutral Valuation: Pricing and Hedging of Financial Derivatives*, Springer, 2013.

[3] T. Björk, *Arbitrage Theory in Continuous Time,* Oxford University Press, 2009.

[4] S.E. Shreve, *Stochastic Calculus for Finance II: Continuous-Time Models,* Vol. 11, Springer, 2004.

Semester IV

MMATH18-401(i): Advanced Group Theory

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The course aims to introduce the learner to the concepts of normal series, composition series and Zessenhaus lemma. A study of solvable groups, nilpotent group and fitting and Frattini subgroup will be conducted and the students will be introduced to free group, presentation of a group and properties of a free group.

Course Learning Outcomes: After doing this course student will be able to

CO1. prove Schreier's refinement theorem and Jordan–Holder theorem and also to prove fundamental theorem of arithmetic using Jordan–Holder theorem.

CO2. prove Hall's theorem, Schur's theorem and Burnside basis theorem.

CO3. identify indecomposable spaces and to prove Krull–Schmidt theorem.

CO4. determine distinct presentations of a group.

Contents:

Unit I: Normal series, composition series, Zessenhaus lemma, Schreier's refinement theorem, JordanHolder theorem.

Unit II: Solvable groups, derived series, supersolvable groups, minimal normal subgroup, Hall's theorem, Hall subgroup, p-complements, central series, Schur's theorem.

Unit III: Nilpotent groups, Fitting subgroup, Jacobi identity, Three subgroup lemma, Frattini subgroup, Burnside basis theorem. Indecomposable groups, Fitting's lemma, KrullSchmidt theorem, Semidirect product.

Unit IV: Free group, Generators and relations of a group, Rank of free group, Projective and injective property of a free group, Free semigroup and representation of a quotient semigroup.

Suggested Readings:

[1] T.W. Hungerford, *Algebra*, Springer-Verlag, 1981.

[2] D.J.S. Robinson, *A Course in the Theory of Groups*, Springer-Verlag, 1996.

[3] J.S. Rose, *A Course on Group Theory*, Dover Publications, 2012.

[4] J.J. Rotman, *An Introduction to the Theory of Groups*, Springer, 1995.

[5] M. Suzuki, *Group Theory-I*, Springer, 2014.

MMATH18-401(ii): Algebraic Number Theory

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The theory of algebraic numbers has its origin in the Fermat's last theorem. In this course we introduce algebraic number fields and their ring of integers and prove that factorization into irreducibles is not always unique in the ring of integers but every ideal is a unique product of prime ideals. We use geometric methods arising in Minkowski's Theorem on convex sets relative to lattices to prove the finiteness of class number of a number field.

Course Learning Outcomes: After studying this course the student will be able to

CO1. classify algebraic number fields, define algebraic integers, ring of integers and integral bases, and calculate norms and traces. It would be possible to determine the integral bases and ring of integers of quadratic and p-th cyclotomic fields.

CO2. construct examples of non-unique factorization domains and apply the unique factorization of certain ring of integers of number fields to solve some Diophantine equations.

CO3. prove uniqueness of factorization of ideal of ring of integers of a number fields in terms of prime ideals. It also leads to deduction of Two-Squares and Four-Squares theorem using Minkowski's theorem on convex sets.

CO4. to visualize ideals of ring of integers as lattices and develop tools to prove the finiteness of class-group.

Contents:

Unit I: Algebraic numbers, Conjugates and discriminants, Algebraic integers, Integral bases, Norms and traces, Rings of algebraic integers, Quadratic and cyclotomic fields.

Unit II: Trivial factorization, Factorization into irreducibles, Examples of non-unique factorization into irreducibles, Prime factorization, Euclidean domains, Euclidean quadratic fields, Consequence of unique factorization the Ramanujan–Nagell theorem.

Unit III: Prime factorization of ideals, Norm of an ideal, Non-unique factorization in cyclotomic fields, Lattices, Quotient torus, Minkowski's theorem, Two-Squares theorem, Four-Squares theorem.

Unit IV: Space L^{st}, Class-group and class-number, Finiteness of the class-group, factorization of a rational prime, Minkowski's constants, Some class-number calculations.

Suggested Readings:

1] Ş. Alaca and K.S. Williams, *Introductory Algebraic Number Theory*, Cambridge University Press, Cambridge, 2003.

[2] K. Ireland and M. Rosen, *A Classical Introduction to Modern Number Theory*, Second Edition, Springer-Verlag, 1990.

[3] S. Lang, *Algebraic Number Theory*, Springer, 1994.

[4] D.A. Marcus, *Number Fields*, Springer, 2018.

[5] I. Stewart and D. Tall, *Algebraic Number Theory and Fermat's Last Theorem*, Fourth Edition, CRC Press, Taylor & Francis, 2016.

MMATH18-401(iii): Simplicial Homology Theory

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The purpose of this course is to learn the technique of computing a sequence of simplicial homology groups associated with compact triangulable spaces and to prove the topological invariance of simplicial homology groups (up to homotopy). The tools of homology theory are applied to prove a few classical applications like fixed point theorems, invariance of dimension, Euler's formula, etc.

Course Learning Outcomes: After studying this course student will be able to

CO1. identify hyperplanes, simplexes and finite simplicial complexes as subsets of a Euclidean space.

CO2. learn the idea of compact triangulable spaces as geometric carriers of finite simplicial complexes (polyhedra).

CO3. learn the use of homological algebra to associate simplicial homology groups with triangulable spaces and illustrate it by computing simplicial homology groups of some well-known compact polyhedral.

CO4. understand the topological invariance of simplicial homology groups (up to homotopy).

CO5. prove important applications of simplicial homology theory like invariance of dimension, Euler's formula, Lefschetz and Brouwer's fixed point theorems, etc.

Contents:

Unit I: Geometric simplexes, Geometric complexes and polyhedra, Barycentric subdivision, Simplicial maps, Simplicial approximation of continuous maps between two polyhedral.

Unit II: Orientation of geometric complexes, Chain complexes, Simplicial homology groups, Structure of homology groups, Relative homology groups, Computation of homology groups, Homology groups of n-sphere.

Unit III: Chain mappings, Chain derivation, Chain homotopy, Contiguous maps, Homomorphism induced by continuous maps between two polyhedra, Functorial property of induced homomorphisms, Topological and homotopy invariance of homology groups.

Unit IV: Euler–Poincare theorem and Euler's formula, Invariance of dimension, Brouwer's fixed point theorem, Degree of self-mappings of S^n, Brouwer's degree theorem, Existence of eigen values, Lefschetz fixed point theorem.

Suggested Readings:

[1] M. K. Agoston, *Algebraic Topology: A First course,* Marcel Dekker, 1976.
[2] M.A. Armstrong, *Basic Topology*, Springer, 1983.
[3] F.H. Croom, *Basic Concepts of Algebraic Topology*, Springer, 1978.
[4] S. Deo, *Algebraic Topology - A Primer*, Second Edition, Hindustan Book Agency, 2018.

MMATH18-401(iv): Theory of Noncommutative Rings

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: To give students an understanding of Wedderburn–Artin theory of semisimple rings, Jacobson's general theory of radicals, prime and semiprime rings, primitive and semiprimitive rings.

Course Learning Outcomes: After studying this course the student will be able to

CO1. know modern ring theory, building on module theory, chain conditions, tensor products and some conceps of commutative algebra.

CO2. know an extensive variety of rings, including free rings, Weyl algebra, Hilbert twist, triangular ring etc.

CO3. understand module theoretic definition of semisimple rings and how it leads to the Wedderburn–Artin structure theorem on their complete classification.

CO4. know Jacobson's general theory of radicals, semiprime rings, primitive and semiprimitive rings, local rings.

CO5. understand the significance of modern ring theory as a meeting point for various branches of mathematics – group theory, representation theory, algebraic geometry, homological algebra.

Contents:

Unit I: Basic terminology and examples, Dedekind-finite rings, Opposite rings, Quaternions, Free *k*-rings, Rings with generators and relations, Weyl algebra, Formal power series ring, Division ring of formal Laurent series, Hilbert's twist ring, Differential polynomial rings, Triangular rings, Example of one-sided Noetherian and Artinian rings.

Unit II: Semisimple rings, Structure of semisimple rings, Wedderburn–Artin's theorem, Structure theorem of simple Artinian rings, Example of non-Artinian simple rings, Jacobson radical, Hopkins–Levitzki theorem, Nakayama's lemma.

Unit III: Prime radical, Prime and semiprime rings, Levitzki theorem, Levitzki radical.

Unit IV: Structure theorem of primitive rings, Density theorem, Subdirect products, Commutativity theorems.

Suggested Readings:

[1] I.N. Herstein, *Noncommutative Rings*, The Mathematical Association of America, 2005.
[2] T.W. Hungerford, *Algebra*, Springer-Verlag, New York, 1981.
[3] T.-Y. Lam, *A First Course in Noncommutative Rings*, Springer, 2001.
[4] L.H. Rowen, *Ring Theory*, Student Edition, Academic Press, 1991.

MMATH18-402(i) Abstract Harmonic Analysis

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: This is an introduction to that part of analysis on locally compact groups that can be done with minimal assumptions on the nature of the group. As a generalization of classical Fourier analysis, this abstract theory creates a foundation for a great deal of modern analysis.

Course Learning Outcomes: After studying this course the student will

CO1. Be familiar with Banach algebras and their representations.

CO2. have studied relations between representations of locally compact groups and representations of group algebras.

CO3. have studied representation theory of compact groups.

CO4. know the notions of orthonormal basis of space of square integrable functions on compact groups.

CO5. know positive definite functions, Bochner theorem, semi direct product of groups and their representation.

Contents:

Unit I: Introduction to representation theory of involutive Banach algebra, Uitary representation of locally compact groups, Gelfand−Raikov theorem.

Unit II: Unitary representation of compact groups, Schur's lemma, Orthogonality relations, Characters of finite dimensional representations, Weyl−Peter theorem.

Unit III: Representation of some special groups $SU(2)$, Lorentz group, Group of linear transformations of .

Unit IV: Convolution of bounded regular complex measures, Convolution of Banach algebra $M(G)$, Fourier−Stieltjes transform, Positive definite functions, Bochner's theorem.

Suggested Readings:

[1] J.M.G. Fell and R.S. Doran, *Representation of *-Algebras, Locally Compact Groups and Banach * Algebraic Bundles*, Vol I, II, Academic Press, 1988.

[2] G.B. Folland, *A Course in Abstract Harmonic Analysis*, Second Edition, CRC Press, Taylor & Francis, 2015.

[3] E. Hewitt and K.A. Ross, *Abstract Harmonic Analysis*, *Vol I Structure of Topological Groups Integration Theory Group Representations*, Springer, 1994.

[4] W. Rudin, *Fourier Analysis on Groups*, Dover Publications, 2017.

MMATH18-402 (ii): Frames and Wavelets

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The course aims to introduce a flexible system which provide stable reconstruction and analysis of functions (signals) and to construction of variety of orthonormal bases by applying operators on a single wavelet function.

Course Learning Outcomes: After studying this course the student will be able to
CO1. understand approximation of functions (signal) by frame theory.
CO2. understand frame transform in sampling theory.
CO3. explain the applications of frames in stable analysis and decompositions of functions.
CP4. explain the applications of wavelets in the construction of orthonormal bases by wavelets.

Contents:

Unit I: Finite frames, Canonical reconstruction formula, Frames and matrices, Similarity and unitary equivalence of frames, Frame bounds and frame algorithms.

Unit II: Frames and Bessel sequences in infinite dimensional Hilbert spaces, Frame sequence, Gram matrix, Frames and operators, Characterization of frames, Dual frames, Tight frames, Continuous frames, Frames and signal processing, Tight frames and dual frame pairs.

Unit III: Riesz bases, Frames versus Riesz bases, Conditions for a frame being a Riesz basis, Frames containing a Riesz basis, Bases in Banach spaces, Limitations of bases.

Unit IV: Wavelets, Haar wavelets, Basic properties of the Haar scaling function, Haar decomposition and reconstruction algorithms, Daubechies wavelets, Wavelet bases, Scaling function.

Suggested Readings:
[1] A. Boggess and F.J. Narcowich, *A First Course in Wavelets with Fourier Analysis*, Second Edition, John Wiley& Sons, 2009.
[2] O. Christensen, *An Introduction to Frames and Riesz Bases*, Second Edition, Birkhäuser, 2016.
[3] D. Han, K. Kornelson, D. Larson and E. Weber, *Frames for Undergraduates*, American Mathematical Society, Student Mathematical Library, Volume 40, 2007.
[4] D.F. Walnut, *An Introduction to Wavelet Analysis*, Springer, 2013.

MMATH18-402 (iii): Operators on Hardy Hilbert Spaces

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The course aims to familiarize the learner with basic properties of Hardy Hilbert spaces and some linear operators defined on such spaces. Properties of Toeplitz, Hankel and Composition operators defined on Hardy Hilbert spaces are studied in detail.

Course Learning Outcomes: After studying this course the student will be able to
CO1. define the Hardy Hilbert space and understand its main properties.
CO2. understand the relation between a function in the Hardy Hilbert space and its boundary values.
CO3. understand the characterizations of invariant and reducing subspaces of the shift operators.
CO4. compute matrices for standard operators on Hilbert spaces with respect to a given orthonormal basis.
CO5. define Toeplitz operators, identify them, and understand their properties.
CO6. define Hankel operators, identify them and understand their properties.
CO7. know what composition operators are and be familiar with their basic properties.

Contents:

Unit I: Hardy Hilbert Space, Basic Definitions and properties, Unilateral shift and factorisation of spectral structure, Functions, Shift operators, Invariant and reducing subspaces, Inner and outer factorisation, Blaschke factors, Singular inner functions, Outer functions.

Unit II: Toeplitz operators, Basic properties of Toeplitz operators, Spectral structure.

Unit III: Hankel operators, Bounded Hankel operators, Hankel operators of finite rank, Compact Hankel operators, Self adjointness and normality of Hankel operators, Relation between Hankel and Toeplitz operators.

Unit IV: Definition and basic properties of composition operators, Spectral properties.

Suggested Readings:

[1] R.A. Martinez-Avedano and P. Rosenthal, *An Introduction to the Operators on Hardy-Hilbert Space*, Springer, 2007.

[2] R.G. Douglas, *Banach Algebra Techniques in Operator Theory*, Second Edition, Springer, 1998.

[3] N.K. Nikolski, *Operators, Functions and Systems: An Easy Reading*, *Volume 1, Hardy, Hankel, and Toeplitz*, American Mathematical Society, 2002.

MMATH18-403(iv): Theory of Unbounded Operators

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The primary objective of this course is to understand the notion of unbounded operators and get familiar with simple examples of such operators. The importance of such operators in applications, particularly for solving differential equations and the role of operator semigroups in this context has to be made clear to the learner.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand the basics of unbounded operators and identify when a given linear operator is bounded or unbounded.

CO2. undrstand closed and closable linear operators on Banach spaces.

CO3. be able to compute adjoints of unbounded linear operators.

CO4. understand spectral properties of self-adjoint operators, multiplication and differentiation operators.

CO5. understand the basic theory of semigroups and their generators.

CO6. understand and appreciate the role unbounded operators and semigroups play in applications, particularly in studying solutions of partial differential equations.

Contents:

Unit I: Unbounded linear operators and their Hilbert adjoints, Hellinger−Toeplitz theorem, Hermitian, Symmetric and self-adjoint linear operators.

Unit II: Closed linear operators, Closable operators and their closures on Banach spaces, Cayley transform, Deficiency indices.

Unit III: Spectral properties of self-adjoint operators, Multiplication and differentiation operators and their spectra.

Unit IV: Semigroup of bounded linear operators, Uniformly continuous and strongly continuous semigroups, Generator of a semigroup, Hille−Yosida theorem, Dissipative operators, Lumer−Phillip theorem, Properties of dissipative operators, Group of bounded linear operators, Stones theorem.

Suggested Readings:

[1] S. Goldberg, *Unbounded Linear Operators: Theory and Applications*, Dover Publications, 2006.

[2] E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, India, 2006.

[3] A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, Springer, 1983.

[4] M. Schechter, *Principles of Functional Analysis*, Second Edition, American Mathematical Society, 2001.

MMATH18-403(i): Calculus on $\mathbb{R}^n$

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: Introduce differentiation of vector valued functions on $\mathbb{R}^n$ and their properties, integration of functions over a rectangle in $\mathbb{R}^n$, change of variables, partition of unity, concepts of differential forms and simplexes and chains leading to the Stokes' theorem.

Course Learning Outcomes: After studying this course the student will be able to

CO1. check differentiability of vector valued functions on $\mathbb{R}^n$, understand the relation between directional derivative and differentiability, apply chain rule, mean value theorems, inverse and implicit function theorems.

CO2. understand higher order derivatives and be able to apply Taylor's formulas with integral remainder, Lagrange's remainder and Peano's remainder.

CO3. learn the concepts of integration over a *k*-cell, primitive mappings, partition of unity, change of variables and be able to work out examples.

CO4. grasp the abstract concept of differential forms and their basic properties including differentiation and change of variables.

CO5. learn the concepts of simplexes and chains and integration of differential forms on such objects, understand and be able to apply Stokes' theorem.

Contents:

Unit I: The differentiability of functions from $\mathbb{R}^n$ to $\mathbb{R}^m$, partial derivatives and differentiability, Directional derivatives and differentiability, Chain rule, Mean value theorems, Inverse function theorem and implicit function theorem.

Unit II: Derivatives of higher order, Taylor's formulas with integral reminder, Lagrange's reminder and Peano's reminder, Integration over a *k*-cell, Primitive mappings, Partition of unity, Change of variables.

Unit III: Introduction to differential forms on $\mathbb{R}^n$, Basic properties of differential forms, Differentiation of differential forms, Change of variables in differential forms.

Unit IV: Simplexes and chains, Integration of differential forms, Stokes' theorem.

Suggested Readings:

[1] M. Giaquinta and G. Modica, *Mathematical Analysis: An Introduction to Functions of Several Variables*, Birkhäuser, 2009.

[2] J.R. Munkres, *Analysis on Manifolds*, CRC Press, Taylor & Francis, 2018.

[3] W. Rudin, *Principles of Mathematical Analysis*, 3rd Edition, Mc Graw Hill, 1986.

[4] M. Spivak, *Calculus on Manifolds: A Modern Approach to Classical Theorems of Advanced Calculus*, Taylor & Francis, 2018.

MMATH 14-403(ii): Differential Geometry

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The primary objective of this course is to understand the notion of level sets, surfaces as solutions of equations, geometry of orientable surfaces, vector fields, Gauss map, geodesics, Weingarten maps, line integrals, parametrization of surfaces, areas, volumes and Gauss–Bonnet theorem.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand the concepts of graphs, level sets as solutions of smooth real valued functions vector fields and tangent space.

CO2. comfortably familiar with orientation, Gauss map, geodesic and parallel transport on oriented surfaces.

CO3. learn about linear self-adjoint Weingarten map and curvature of a plane curve with applications in geometry and physics.

CO4. know line integrals, be able to deal with differential forms and calculate arc length and curvature of surfaces.

CO5. deal with parametrization and be familiar with well-known surfaces as equations in multiple variables, able to find area and volumes.

CO6. study surfaces with boundary and be able to solve various problems and the Gauss–Bonnet theorem.

Contents:

Unit I: Graph and level sets, vector fields, tangent spaces

Unit II: Surfaces, orientation, the Gauss map, geodesics, parallel transport.

Unit III: Weingarten map, curvature of plane curves, arc length and line integrals, curvature of surfaces.

Unit IV: Parametrized surfaces, surface area and volume, surface with boundary, the GaussBonnet theorem**.**

Suggested Readings:

[1] W. Kühnel, *Differential Geometry, Curves-Surfaces-Manifolds*, Third Edition, American Mathematical Society, 2013.

[2] A. Mishchenko and A. Formentko, *A Course of Differential Geometry and Topology*, Mir Publishers Moscow, 1988.

[3] A. Pressley, *Elementary Differential Geometry*, Springer, India, 2004.

[4] J.A. Thorpe, *Elementary Topics in Differential Geometry*, Springer, India, 2004.

MMATH18-403(iii): Topological Dynamics

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The objective of the course is provide a strong background of Topological Dynamical Systems including their applications and to study some useful and interesting dynamical properties like expansivity, shadowing and transitivity with supporting examples and results from Symbolic and Topological dynamics including the celebrated Sarkovskii's theorem. The course aims at providing practice of reading and understanding research papers in this area.

Course Learning Outcomes: After studying this course the student will be able to

CO1. know interesting examples of Dynamical Systems, Topological conjugacy, Types of orbits, Stable sets, Omega and alpha limit sets.

CO2. have studied celebrated Sarkovskii's theorem, Dynamics of logistic functions, shifts and subshifts.

CO3. have studied dynamical properties like expansivity and transitivity, results regarding existence/non-existence, product, subspace etc. and characterization of such maps.

CO4. know about the maps possessing shadowing properties.

CO5. have studied topological Anosov maps and topological stability.

Contents:

Unit I: Definition and examples (including real life examples), Orbits, Types of orbits, Topological conjugacy and orbits, Phase portrait - Graphical analysis of orbit, Periodic points and stable sets, Omega and alpha limit sets and their properties.

Unit II: Sarkovskii's theorem, Dynamics of Logistic Functions, Shift spaces and subshift, Subshift of finite type, Subshift represented by a matrix.

Unit III: Definition and examples of expansive homeomorphisms, Properties of expansive homeomorphisms, Non-existence of expansive homeomorphism on the unit interval and unit circle, Generators and weak generators, Generators and expansive homeomorphisms.

Unit IV: Converging semiorbits for expansive homeomorphisms, Definition and examples of maps shadowing property, properties of shadowing property, Topological Stability, Anosov maps and topological stability.

Suggested Readings:

[1] N. Aoki and K. Hiraide, *Topological Theory of Dynamical Systems: Recent Advances*, North Holland Publications, 1994.

[2] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press, 2004.

[3] D.C. Hanselman and B. Littlefield, *Mastering MATLAB*, Pearson, 2012.

[4] D. Lind and B. Marcus, *An Introduction to Symbolic Dynamics and Coding*, Cambridge University Press, 1996.

[5] C. Robinson, *Dynamical Systems, Stability, Symbolic Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 1998.

[6] J. de. Vries, *Elements of Topological Dynamics*, Springer, 1993.

[7] J. Dugundji, *Topology*, Allyn and Bacon Inc., Boston, 1978.

MMATH18-404 (i): Advanced Fluid Dynamics

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: To prepare a foundation for advanced study of fluid motion in dimensions of compressible fluid, magnetohydrodynamics and boundary layer theory. Develop concept, models and techniques which enable to solve the problems and help in research in these broad areas.

Course Learning Outcomes: After studying this course the student will be able to

CO1. know about the basics of first and second law of thermodynamics, internal energy, specific heats and concept of entropy, different form of energy equations and dimensional analysis.

CO2. know about compressibility in real fluids, the elements of wave motion, sound wave, shock wave, their formation, properties and elementary analysis.

CO3. understand the interaction between hydrodynamic process and electromagnetic phenomena in term of Maxwell electromagnetic field equation.

CO4. formulate the basic equations of motion in inviscid and viscous conducting fluid flow and be familiar with the Alfven's wave and magneto-hydrodynamic wave**.**

CO5. know the concepts of boundary layer, boundary layer equations and their solutions with different concept and measurement of boundary layer thickness.

Contents:

Unit I: Thermodynamics and dimensional analysis, Equation of state of a substance, First law of Thermodynamics, Internal energy and specific heat of gas, entropy, Second law of thermodynamics, Energy equation, Nondimensionalizing the basic equations of incompressible viscous fluid flow, Non-dimensional numbers.

Unit II: Gas Dynamics, Compressibility effects in real fluids, Elements of wave motion in a gas, Speed of sound, Basic equation of one-dimensional compressible flow, Subsonic, sonic and supersonic flows, Isentropic gas Flow, Flow through a nozzle, Normal shock wave, oblique shock wave and their elementary analysis.

Unit III: Magnetohydrodynamics, Concept, Maxwell' electromagnetic field equations, Equation of motion of a conducting fluid, MHD approximations, Rate of flow of charge, Magnetic Reynolds number and Magnetic field equation, Alfven's theorem, Magnetic body force, Magnetohydrodynamic wave.

Unit IV: Boundary layer theory, Concept of Boundary Layer, Boundary layer thickness, Boundary layer equations for two-dimensional incompressible flow, Boundary layer along a flat plate, General properties of the boundary-layer equations, Dependence on Reynolds number, Similar solutions, Momentum and energy integral equations for the boundary layer.

Suggested Readings:

[1] F. Chorlton, *Text Book of Fluid Dynamics*, CBS Publisher, 2005.
[2] A. Jeffrey, *Magnetohydrodynamics*, Oliver & Boyd, 1966.
[3] H. Schlichting, K. Gersten, *Boundary Layer Theory*, 8th Edition, Springer, 2003.
[4] G.B. Witham, *Linear and Nonlinear Waves*, John Wiley & Sons, 1999.

MMATH18-404(ii) Computational Methods for PDEs

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: The aim of this course is to introduce various numerical methods especially finite difference schemes for the solution of partial differential equations along with analyzing them for consistency, stability and convergence.

Course Learning Outcomes: After studying this course the student will be able to
CO1. use discretization methods for solution of PDEs using finite difference schemes.
CO2. analyze the consistency, stability and convergence of a given numerical scheme.
CO3. apply various iterative techniques for solving system of algebraic equations.
CO4. know the basics of finite element methods for the numerical solution of PDEs.
CO5. construct computer programme using some mathematical software to test and implement numerical schemes studied in the course.

Contents:

Unit I: Finite difference methods for 2D and 3D elliptic boundary value problems (BVPs) of second approximations, Finite difference approximations to Poisson's equation in cylindrical and spherical polar coordinates, Solution of large system of algebraic equations corresponding to discrete problems and iterative methods (Jacobi, Gauss–Seidel and SOR), Alternating direction methods.

Unit II: Different 2- and 3-level explicit and implicit finite difference approximations to heat conduction equation with Dirichlet and Neumann boundary conditions, Stability analysis, compatibility, consistency and convergence of the difference methods, ADI methods for 2- & 3-D parabolic equations, Finite difference approximations to heat equation in polar coordinates.

Unit III: Methods of characteristics for evolution problem of hyperbolic type, explicit and implicit difference schemes for first order hyperbolic equations in 1D and 2D dimension and their stability and consistency analysis, System of equations for first order hyperbolic equations.

Unit IV: Finite element methods for second order elliptic BVPs, Finite element equations, Variational problems, Triangular and rectangular finite elements, Standard examples of finite elements, Finite element methods for parabolic initial and boundary value problems.

Suggested Readings:

[1] A.J. Davies, *The Finite Element Method: An Introduction with Partial Differential Equations*, Second Edition, Oxford University Press, 2011.
[2] C. Johnson, *Numerical Solution of Partial Differential Equations by the Finite Element Methods*, Dover Publications, 2009.
[3] K.W. Morton and D.F. Mayers, *Numerical Solution of Partial Differential Equations*, Second Edition, Cambridge University Press, 2011.
[4] J.C. Strikwerda, *Finite Difference Schemes & Partial Differential Equations*, Second Edition, SIAM, 2004.
[5] J.W. Thomas, *Numerical Partial Differential Equations: Finite Difference Methods*, Springer, 2013.
[6] J.W. Thomas, *Numerical Partial Differential Equations: Conservation Laws and Elliptic Equation*s, Springer-Verlag, Berlin, 1999.

MMATH18-404(iii): Dynamical Systems

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: This course prepares foundation for understanding discrete and continuous systems with case studies, is devoted to a study of nonlinear systems of ordinary differential equations and dynamical systems. The concept, models and techniques included in this course enables to understand the real world problems and stability of the systems are analyzed along with the chaotic dynamic behavior of models by understanding bifurcations.

Course Learning Outcomes: After studying this course the student will be able to

CO1. understand formulation of mathematical models with the stability analysis near the equilibrium points.

CO2. know how the concept of phase portraits help to analyze mathematical model graphically.

CO3. describe the qualitative behavior of the solution set of a given system of differential equations including the invariant sets and limiting behavior of the dynamical system or flow defined by the system of differential equations.

CO4. understand how different bifurcations explain the chaotic behavior of the system.

Contents:

Unit I: First order continuous autonomous systems–some terminology, classification of fixed points of autonomous systems, attractors and repellors, Case study, population growth, Second order continuous autonomous systems – autonomous second order systems, Constant coefficient equations, Phase curves and fixed points, Classification of fixed points of linear systems.

Unit II: Nonlinear Systems – flow, Linearization, Stable manifold theorem, Hartman–Grobman theorem, Stability and Liapunov functions, Saddles, Nodes, Foci and centres, Nonhyperbolic Critical points, Case study, Interacting species with programming.

Unit III: Discrete Systems – examples of discrete systems, Some terminology, Linear discrete systems, Non-linear discrete systems, Quadratic maps.

Unit IV: Bifurcations in one-dimensional flows – Introduction, saddle-node bifurcation, Transcritical bifurcation, Pitchfork bifurcation, Bifurcation in two-dimensional flows – saddle-node, Transcritical, Pitchfork bifurcations and Hopf bifurcations.

Suggested Readings:

[1] R.L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*, CRC Press, Taylor & Francis, 2018.

[2] M.W. Hirsch, S. Smale, R.L. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*, Third Edition, Academic Press, 2013.

[3] S.H. Strogatz, *Nonlinear Dynamics and Chaos*, Second Edition, CRC Press, Taylor & Francis, 2018.

[4] L. Perko, *Differential Equations and Dynamical Systems,* Third Edition, Springer Verlag, 2001.

MMATH18-404(iv): Optimization Techniques and Control Theory

Total Marks: 100 (70 End Semester Examination + 30 Internal Assessment)

Course Objectives: One of the objective of the course is to develop the conjugate duality theory and deal with some numerical techniques to solve a nonlinear problem. Further, the course aims to study dynamic programming approach to solve different types of problems and to study optimal control problems.

Course Learning Outcomes: After studying this course the student will be able to

CO1. have studied notions of sub-gradients and directional derivative for nondifferentiable functions.

CO2. understand the use of conjugate functions to develop the theory of conjugate duality.

CO3. know numerical methods like gradient descent method, gradient projection method, Newton's method and conjugate gradient method.

CO4. deal with dynamic programming approach to solve some problems including stage coach problem, allocation problem and linear programming problem.

CO5. know both classical and modern approaches in the study of optimal control problems.

Contents:

Unit I: Extended real valued functions, Proper convex functions, Subgradients, Directional derivatives

Unit II: Conjugate functions, Dual convex programs, Optimality conditions and Lagrange multipliers, Duality and optimality for standard convex programs, Gradient descent method, Gradient projection method.

Unit III: Newton's method, Conjugate gradient method, Dynamic programming, Bellman's principle of optimality, Allocation problem, Stage coach problem.

Unit IV: Optimal control problem and formulations, Variational approach to the fixed-time free endpoint problem, Pontryagin's maximum principle, Dynamic programming and Hamilton−Jacobi−Bellman equation.

Suggested Readings:

[1] M. Avriel, *Nonlinear Programming: Analysis & Methods*, Dover Publications, New York, 2003.

[2] O. Güler, *Foundations of Optimization*, Springer 2010.

[3] F.S. Hillier, G.J. Lieberman, P. Nag and P. Basu, *Introduction to Operations Research*, Tata McGraw-Hill, 2012.

[4] D. Liberzon, *Calculus of Variations and Optimal Control Theory: A Concise Introduction*, Princeton University Press, 2012.

MMATH18-405(i): Cryptography

Total Marks: 50 (35 End Semester Examination + 15 Internal Assessment)

Course Objectives : This course aims at familiarising the students to cryptography. Classical ciphers and their cryptanalysis have been discussed. Linear feedback shift registers have been studied. RSA and Diffie Hellman key exchange have been described.

Course Learning Outcomes: After studying this course the student will

CO1. have been introduced to the concept of secure communication and fundamentals of cryptography.

CO2. know classical ciphers such as Vigenere Cipher and Hill Cipher.

CO3. have insight into DES and AES.

CO4. be familiar with secure random bit generator and linear feedback shift register sequences.

CO5. know of RSA, attacks on RSA, Diffie−Hellman key exchange and ElGamal, public key cryptosystem.

Contents:

Unit I: Secure communication, Symmetric and asymmetric cryptosystems, Block and Stream Ciphers, Affine linear Block Ciphers and their cryptanalysis, Perfect Secrecy, Vernam One time pad, Secure random bit generator, Linear feedback shift registers.

Unit II: Feistel Cipher, DES, Modes of DES, AES, RSA, Diffie Hellman key exchange, Elgamal public key cryptosystem.

Suggested Readings:

[1] J.A. Buchmann, *Introduction to Cryptography*, Second Edition, Springer 2003.

[2] D.R. Stinson, *Cryptography - Theory and Practice*, CRC Press, Taylor & Francis, 2005.

[3] W. Trappe and L.C. Washington, *Introduction to Cryptography: With Coding Theory*, Pearson, 2006.

MMATH18-405(ii): Support Vector Machines

Total Marks: 50 (35 End Semester Examination + 15 Internal Assessment)

Course Objectives: The aim of the course is to introduce optimization based perspective of the support vector machines (SVM) which has proven effective and promising techniques for data mining. The focus is to deal with the two main components of SVMs namely, classification problems and regression problems based on linear and nonlinear separation.

Course Learning Outcomes: After studying this course the student will be able to

CO1. have a brief understanding of convex optimization.

CO2. have an idea of basic linear support vector classification and linear support vector regression.

CO3. have acquired the understanding of kernel theory, which is the foundation for solving nonlinear problems, and would have become aware of the general classification and regression problems.

Contents:

Unit I: Convex Programming in Euclidean Space, Duality Theory, Optimality Conditions, Linear Classification, Presentation of Classification Problems, Maximal Margin Method and Linearly Separable Support Vector Classification, Maximal Margin Method and Linear C-Support Vector Classification.

Unit II: Regression Problems, Linear Regression Problem and Hard Band Hyperplane, Linear Classification and Hard Band Hyperplane, Optimization Problem of Constructing a Hard Band Hyperplane, Linear Hard Band Support Vector Regression, Linear Support Vector Regression, Classification Machine Based on Nonlinear Separation, Regression Machine Based on Nonlinear Separation, Properties of Kernels, Construction of Kernels, Support Vector Classification, Support Vector Regression, Flatness of Support Vector Machines.

Suggested Readings:

[1] N. Cristianini, J. Shawe-Taylor, *An Introduction to Support Vector Machines and Other Kernel-Based Learning Methods*, Cambridge University Press, 2000.

[2] N. Deng, Y. Tian and C. Zhang, *Support Vector Machines: Optimization Based Theory, Algorithms, and Extensions*, CRC Press, Taylor & Francis, 2012.

[3] M. Kubat, *An Introduction to Machine Learning*, Springer, 2015